N Nash equilibria

Pablo and Juan discover an irregularity in the public sector. Although each of them supports reporting this kind of behavior, they would prefer the report to be made by the other. However, the worst situation for both is that the report is not made at all. Both must make a decision (report or not) without knowing what the other will do.

1. Represent the game in normal form.
2. Determine the Nash equilibria.
3. Assume that in reality there was not much discretion in carrying out the irregularity and that all employees know what happened. Let n be the number of employees in the area. Find the Nash equilibria.

Solution

1. We see that there are two players (Pablo and Juan, or player 1 and player 2). There are 2 actions, report or not report. And given the context, it is a simultaneous game with complete information.

   Before representing the game in normal form, we use the information from the statement to draw the following conclusions. Report: R, not report: NR

   - $U_1(NR, R) > U_1(R, NR)$ Player 1 (Pablo) prefers the other to report rather than him reporting.
   - $U_1(R, NR) > U_1(NR, NR)$ He prefers to report himself rather than no one reporting.

   We will go from the particular to the general, then we propose a matrix consistent with what we have seen previously.

		Juan	
		R	NR
Pablo	R	$5; 5$	$5; 10$
	NR	$10; 5$	$1; 1$

   We see that each one wants the other to report and the worst situation is when no one reports.

   If Juan reports, Pablo would prefer not to report, we mark 10. If Juan does not report, Pablo would prefer to report, we mark 5. On the other hand, the same reasoning applies. If Pablo reports, Juan would prefer not to report, we mark 10, if Pablo chooses not to report, Juan would prefer to report, we mark 5.

		Juan	
		R	NR
Pablo	R	$5; 5$	$5;10$
	NR	$10;5$	$1; 1$

   We see that each one wants the other to report and the worst situation is when no one reports.

2. The Nash equilibria are then 2:

   - Pablo does not report and Juan reports
   - Juan does not report and Pablo reports

   ## Generalizing

   The previous game can be generalized as follows:

		Juan	
		R	NR
Pablo	R	$x_1; y_1$	$x_2; y_2$
	NR	$x_3; y_3$	$x_4; y_4$

   Where $x_3 > x_2$, $x_2 > x_4$, $y_2 > y_1$, and $y_3 > y_4$.

3. As always, a good option to think about this case is to look at the extremes: No one reports, everyone reports. And then move on to other cases.

   - If everyone reports, there are incentives to deviate, meaning not to report. Because they prefer others to assume the cost of reporting. It is not a Nash equilibrium (NE).
   - If no one reports, there are incentives for someone to report, since they prefer to report the irregularity rather than it going unnoticed. It is not NE.

Other cases

Let's think about what happens in other cases

- If two report and everyone else keeps quiet, there are incentives to deviate, because each would think individually, why should I report if someone else is going to do it? Then it is not NE.
- The same happens with 3 or more.
- If one reports and everyone else keeps quiet, there are no incentives to deviate since those who are quiet have no incentives to report (as someone else is doing it), and the one reporting thinks that if they didn't do it, no one else would.

Conclusion: There are n Nash equilibria. Each one involves one player reporting and all others not reporting.